

Implementation of the Greedy Algorithm for Coloring Graph Based on Four-Color Theorem

Nurul Maulida Surbakti ¹, Fanny Ramadhani ²

¹ Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Negeri Medan, Jl. Willem Iskandar, Medan, 20221, Indonesia

² Computer Science, Faculty of Mathematics and Natural Sciences, Universitas Negeri Medan, Jl. Willem Iskandar, Medan, 20221, Indonesia

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Phone: +62 823-6399-9245
E-mail: amarhsb7@gmail.com

A B S T R A K

Graph theory is an advanced subject of mathematics that can be utilized to resolve issues in science. Graph coloring is one of the most well-known problems for determining the color of the map. The map that will be colored here is one of the 21 sub-districts that cover the Medan regency. In order to color the map, a graph model of the map must first be created. The use of a greedy algorithm is one technique to find a graph's minimal color. The dual graph with 21 vertices and 45 edges will be what we extract from the map. Based on the greedy method that has been used, just four colors—blue, green, red, and yellow—are found as the smallest number of colors, with each city that borders another having a distinct color. The Python computer language is used to get the map coloring results using the greedy technique.

PENDAHULUAN

Graphs and their application in graph coloring can be used to model the issue. One of the most fascinating and complex combinatorial optimization issues in operations research, mathematics, and computer science is the graph coloring problem (GCP) [1]. In general, there is a connection between the graph coloring issue and the subject of cartography. Making maps is the art form of cartography.

In cartography, a map is said to be accurate and good if it can be readily recognized by the reader and has all the necessary components, such as a legend, scale, title, color, and symbols. One of the benefits of color is that it makes it easier for someone to recognize several regions that have direct borders. A map showing the regions that each of these nations owns is colored in various shades. The map is ineffective when there are too many colors. For that, we require a technique for coloring the map with the fewest possible colors.

Let G be a graph. The chromatic number problem in graph theory involves figuring out the minimum number of colors that may be represented on a graph. The set chromatic number of G , which is represented by the symbol $\chi_s(G)$, is the minimum number of colors needed for a set color (G). The chromatic number problem in graph theory involves figuring out the minimum number of colors that may be represented on a graph [2]. Graph coloring theory can be applied to determining the color on a map, which is known as area coloring.

As long as the two adjoining areas are of different colors, planar graph coloring is connected to map coloring. Francis Guthrie first introduced the four-color theorem (1831). The Four-Color Theorem is the theory that a map with more than

four colors cannot be colored (Four-Color Theorem). If G is a planar graph, then $\chi(G) \leq 4$. In the context of graph coloring, the term "greedy algorithm" refers to assigning colors to a string of adjacent $d + 1$, $\chi(G) \leq d + 1$, if d is the greatest degree of any of its vertices.

There are several previous studies that are indirectly related to this research, including: implementation of the greedy algorithm on graph coloring. A map of the Deli Serdang region will be colored in this [3], minimal proof of the four-color theorem [4], and the ant algorithm [5]-[8].

In this paper, we continue to look for Sipayung *et al* [3][9][10]. We focus on the use of the greedy method and the study topic on an analysis of the Four-Color Theorem. The purpose of this research is to use a greedy coloring algorithm on a map of Medan, with the possibility that graph analysis would provide a different map. The four-color theorem provides the basis for the algorithm's implementation.

METHODS

This research method is a case study using 21 sub-districts that cover the Medan regency. The steps for using the greedy coloring method on a map of the city of Medan are as follows.

1. Creating a representation of a city map of Medan.
2. A recapitulation of the vertex degrees for every sub-district in the city of Medan.
3. Choose the colored vertex, especially from the district or vertex with the highest degree.
4. Use the color selection function to choose a color candidate so that no adjacent vertices or districts have the same color. In other words, non-neighboring vertices and districts will use the same color.
5. Examine if the chosen color is fit for use with the feasibility function, especially by concentrating to nearby vertices and districts. If it is not possible, go back to step 2.

Administratively, Medan City consists of 21 sub-districts. Each subdistrict is assigned a vertex. This can be seen in the table below:

Tables 1. Sub-district vertex labels in Medan regency

Distric Name	Vertex	Distric Name	Vertex
Medan Tuntungan	v_1	Medan Helvetia	v_{12}
Medan Johor	v_2	Medan Petisah	v_{13}
Medan Amplas	v_3	Medan Barat	v_{14}
Medan Denai	v_4	Medan Timur	v_{15}
Medan Area	v_5	Medan Perjuangan	v_{16}
Medan Kota	v_6	Medan Tembung	v_{17}
Medan Maimun	v_7	Medan Deli	v_{18}
Medan Polonia	v_8	Medan Labuhan	v_{19}
Medan Baru	v_9	Medan Marelan	v_{20}
Medan Selayang	v_{10}	Medan Belawan	v_{21}
Medan Sunggal	v_{11}		

The vertices of the following map of the municipality of Medan can be assumed to represent sub-districts, and the edges to be any two sub-districts that are next to one other.

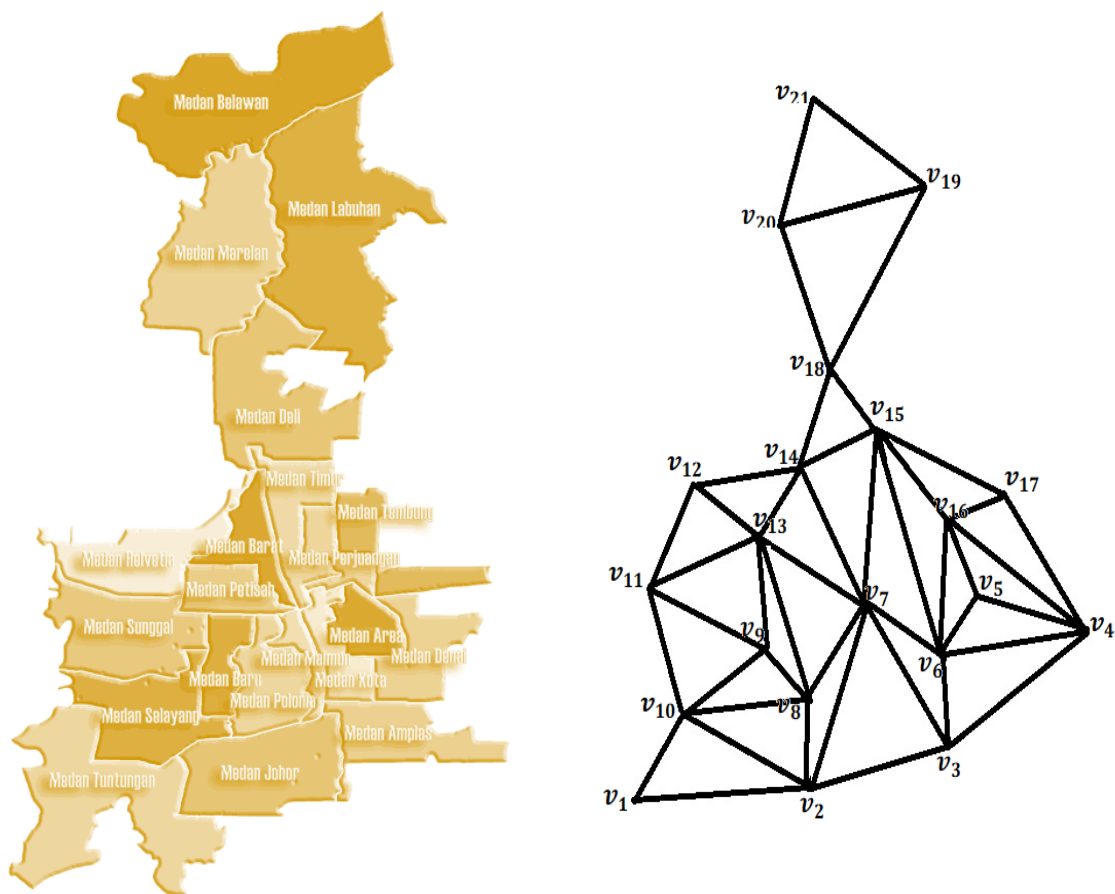


Figure 1. Map and graph model of district map in Medan regency

Some might determine the degree of each point using the graph model in figure 2. The following table displays the degrees of each point in order from highest to lowest level:

Table 2. Degree of each vertex of graph.

Vertex	Degree
v_1	2
v_2	5
v_3	4
v_4	5
v_5	3
v_6	6
v_7	7
v_8	5
v_9	4
v_{10}	5
v_{11}	4

Vertex	Degree
v_{12}	3
v_{13}	6
v_{14}	5
v_{15}	6
v_{16}	5
v_{17}	3
v_{18}	4
v_{19}	3
v_{20}	3
v_{21}	2

RESULT AND DISCUSSION

The greedy algorithm is implemented by a series of steps. The phases of map coloring using Python's greedy method are as follows:

1. Define the colors (Red, Yellow, Blue, and Green) based on the four-color theorem.
2. Use the adjacent matrix to represent the graph. Next, Initiate the name of the vertex with v_i , where $1 \leq i \leq 21$.
3. Count the degree and define the possible color.
4. Sort for arranging the vertex from the largest to the lowest degrees.
5. The primary procedure will search the sortedNode, set the color using the colorDict's list of potential colors, and then store the solution. The color will then be removed from colorDict since it was utilized.
6. Print from theSolution Dict and sort them by the name of the vertex.

Figure 2 displays the results of using the Python programming language to construct the greedy method.

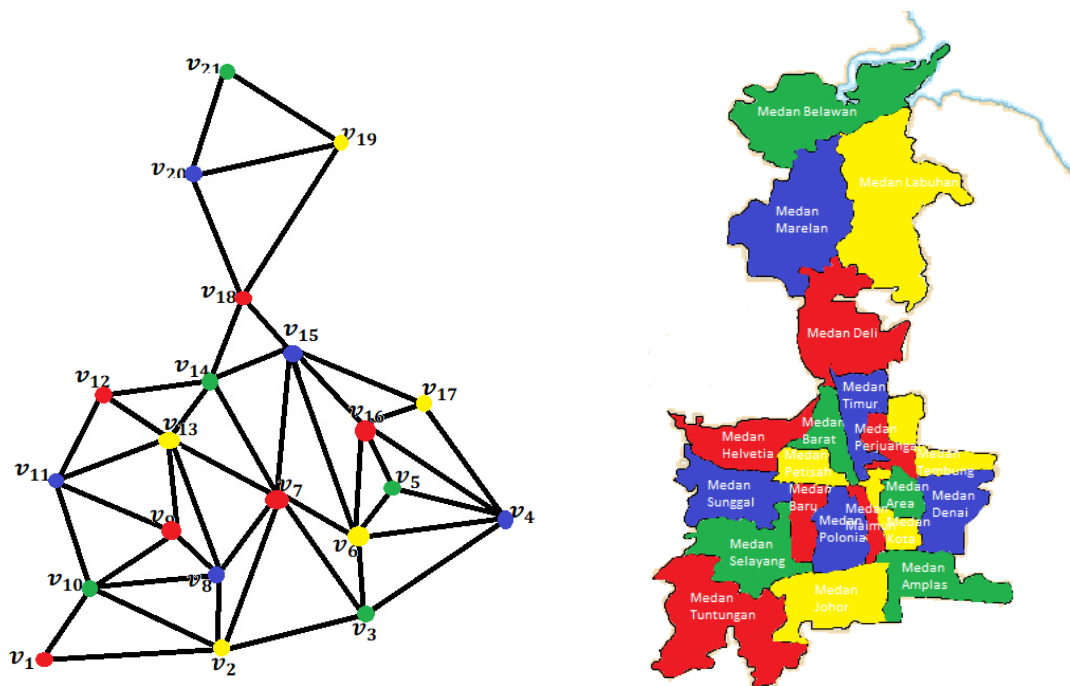


Figure 2. Results of graph coloring and alternative city maps of Medan

CONCLUSIONS

Using the greedy approach, map coloring in Medan regency's sub-districts produced numerous results, including the fact that the region's 21 sub-districts are represented by vertex. One of the options produced by converting the map into a graph model is that 45 edges have been established. Red, yellow, blue, and green are the four colors that make up the C set. The region that is marked in red initially is the Medan Maimun subdistrict since it has the most point degrees. Four colors—red, yellow, blue, and green—were acquired for coloring the sub-district map in the Medan Regency based on the greedy method and the Python computer language.

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